Nonadiabatic coherent evolution of two-level systems under spontaneous decay

F. O. Prado[1], E. I. Duzzioni[1,2], M. H. Y. Moussa[3], N. G. de Almeida[4], and C. J. Villas-Bôas[1]

[1]*Departamento de Física, Universidade Federal de São Carlos, P.O. Box 676, São Carlos, 13565-905, São Paulo, Brazil*

[2]*Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Rua Santa Adélia, 166, Santo André, São Paulo, 09210-170, Brazil*

[3]*Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970, São Carlos, São Paulo, Brazil and*

[4]*Núcleo de Pesquisas em Física, Universidade Católica de Goiás, 74.605-220, Goiânia (GO), Brazil*

Abstract

In this paper we extend current perspectives in engineering reservoirs by producing a time-dependent master equation leading to a nonstationary superposition equilibrium state that can be nonadiabatically controlled by the system-reservoir parameters. Working with an ion trapped inside a nonindeal cavity we first engineer effective Hamiltonians that couple the electronic states of the ion with the cavity mode. Subsequently, two classes of decoherence-free evolution of the superposition of the ground and decaying excited levels are achieved: those with time-dependent azimuthal or polar angle. As an application, we generalise the purpose of an earlier study [Phys. Rev. Lett. **96**, 150403 (2006)], showing how to observe the geometric phases acquired by the protected nonstationary states even under a nonadiabatic evolution.

In the last decade, research activity on the subject of open quantum systems has been mainly devoted to the search for mechanisms to bypass decoherence. Beyond the quest for conditions that weaken the system-reservoir coupling [1], coherence control schemes have been introduced among the protocols for quantum error-correcting codes [2], the existence of decoherence-free subspaces (DFS) in collective systems [3], and dynamical decoupling (DD) methods [4]. More recently, a technique has been presented to reach the same goal as the DD schemes, without interfering directly in the system within the reservoir timescale [5]. We finally mention the engineering reservoir program [6], where a quantum system whose state is to be protected is compelled to engage in additional interactions besides that with the reservoir. Such interactions are carefully engineered to modify the Liouvillian in a specific way that drives the system to an equilibrium with the reservoir. The engineering reservoir has been developed for both trapped ions [7, 8] and atomic two-level (TL) systems [9]. We stress that the engineering reservoir is deeply connected to the engineering Hamiltonian program which has been pursued for quantum information purposes [10]. More recently, under the assumption of a squeezed engineered reservoir, a way to observe the adiabatic geometric phase acquired by a protected state evolving coherently through the adiabatic manipulation of the squeeze parameters of the engineered reservoir has been proposed [11, 12, 13].

In this paper we show how to protect a nonstationary superposition state, broadening the range of the proposed scheme for engineering reservoirs [6]. We present a general recipe to build nonadiabatic coherent evolutions driven by engineered reservoirs. Given this general recipe, the task to achieve a particular nonadiabatic evolution — through a particular engineered reservoir — relies entirely on the engineering Hamiltonian program. Differently from the standard application of the engineering reservoir technique [6, 7, 8], in our model we achieve the nonadiabatic decoherence-free evolution of superposition states, and show how to implement it in a particular system. Working with an ion trapped inside a nonideal cavity, we initially engineer an effective Hamiltonian coupling the electronic levels of the ion with the cavity mode. Assuming a strong decay rate of the cavity field, this effective interaction is employed to build an artificial reservoir, leading to two classes of asymptotic nonstationary superpositions of the ground $|g\rangle$ and decaying excited $|e\rangle$ ionic levels: those with time-dependent azimuthal and with time-dependent polar angles. We stress that by combining both evolutions we can perform any trajectory on the Bloch sphere. As an application of this engineering reservoir technique, we generalize the protocol in Ref. [11, 14], demonstrating how to observe geometric phases acquired by protected nonstationary states even under a nonadiabatic evolution. As a particular case of our model, we also show how to build a quantum memory to protect stationary superpositions of the internal degrees of freedom of the ion.

The main goal of the standard engineering reservoir scheme [6] is to obtain, in the interaction picture, a master equation in the form

$$\dot{\rho} = \frac{\Gamma}{2}\left(2\mathcal{O}\rho\mathcal{O}^{\dagger} - \mathcal{O}^{\dagger}\mathcal{O}\rho - \rho\mathcal{O}^{\dagger}\mathcal{O}\right), \tag{1}$$

where Γ is the effective decay rate of the engineered reservoir which is coupled to the quantum system in a specific way characterized by the time-independent system operator $\mathcal{O}$. The only pure steady state of this system

is the eigenstate $|\psi\rangle$ of the operator $\mathcal{O}$ with null eigenvalue, ensuring that there is no further eigenstate $|\phi\rangle$ of $\mathcal{O}$ such that $\left[\mathcal{O},\mathcal{O}^{\dagger}\right]|\phi\rangle = 0$ [8]. Even considering time-dependent Lindblad operators in master equation (1), the scheme used to protect a given state remains valid, assuming an adiabatic evolution of the reservoir parameters, i. e., the rate of change of operator $\mathcal{O}$, characterized by ϖ, is much smaller than Γ. Consequently, we obtain a nonstationary protected state $|\psi(t)\rangle$, which is the instantaneous eigenstate of $\mathcal{O}$ with null eingenvalue and follows the adiabatic changes of the reservoir parameters [14]. Of course, the fidelity of the protected state in this adiabatic evolution depends on the ratio $\varpi/\Gamma \ll 1$. Next, to remove the adiabatic constraint in the decoherence-free evolution described above, we consider the engineered time-dependent master equation in the interaction picture ($\hbar = 1$)

$$\dot{\rho} = -i\left[H(t),\rho\right] + \frac{\Gamma}{2}\left[2\mathcal{O}(t)\,\rho\mathcal{O}^{\dagger}(t) - \mathcal{O}^{\dagger}(t)\,\mathcal{O}(t)\,\rho - \rho\mathcal{O}^{\dagger}(t)\,\mathcal{O}(t)\right], \tag{2}$$

where the Hermitian Hamiltonian $H(t)$ must be chosen in accordance with the time dependence of the operator $\mathcal{O}(t) = R(t)\,\mathcal{O}_R R^{\dagger}(t)$, with $R(t) = T\exp\left(-i\int_0^t H(t')\,dt'\right)$, T being the time-ordering operator. Note that through the unitary transformation $R(t)$, we recover the time-independent form of the master equation given in (1), in a representation where $\mathcal{O}$ is replaced by $\mathcal{O}_R$. Interestingly, the protected stationary eigenstate $|\psi_R\rangle$ ($\mathcal{O}_R|\psi_R\rangle = 0$), turns out to be a nonstationary state in the original interaction picture, $|\psi(t)\rangle = R(t)|\psi_R\rangle$, whose evolution can be manipulated by means of appropriate engineered Hamiltonian $H(t)$ and reservoir.

Now we show how to implement the ideas discussed above, using a TL trapped ion characterized by the transition frequency ω_0 between the ground $|g\rangle$ and excited $|e\rangle$ states and trap frequency ν. The transition $|g\rangle \leftrightarrow |e\rangle$ is driven by (one or two) classical fields of frequencies ω_ℓ, with coupling strengths Ω_ℓ (with $\ell = 1, 2$), and the ion is made to interact — under the Jaynes-Cummings Hamiltonian and Rabi frequency g — with a cavity mode of frequency ω_a. Within the rotating-wave approximation (RWA), the Hamiltonian modelling the system is given by

$$H = \omega_a a^{\dagger}a + \omega_0\sigma_z/2 + \nu b^{\dagger}b + \left\{g\cos\left(\overrightarrow{k}.\overrightarrow{x}\right)a\sigma_{eg}\,, + \left[\Omega_1\,\mathrm{e}^{i\left(\overrightarrow{k_1}.\overrightarrow{x}+\phi_1-\omega_1 t\right)} + \Omega_2\,\mathrm{e}^{i\left(\overrightarrow{k_2}.\overrightarrow{x}+\phi_2-\omega_2 t\right)}\right]\sigma_{eg} + \mathrm{H.c.}\right\} \tag{3}$$

where $a^{\dagger}$ (a) and $b^{\dagger}$ (b) are the creation (annihilation) operators of the quantized harmonic field and the vibrational mode whose position operator is $\overrightarrow{x} = (b^{\dagger} + b)/\sqrt{2m\nu}\widehat{x}$, m being the ionic mass. The wave vectors $\overrightarrow{k}$, $\overrightarrow{k}_1$, and $\overrightarrow{k}_2$ stand for the cavity mode and the two amplification fields (with dephasings ϕ_1 and ϕ_2), respectively, while $\sigma_{kl} \equiv |k\rangle\langle l|$ (k and l being the states g and e). The vibrational mode is decoupled from the remaining degrees of freedom of our model by assuming the wave vectors $\overrightarrow{k}$, $\overrightarrow{k}_1$, and $\overrightarrow{k}_2$ to be perpendicular to $\overrightarrow{x}$. Under this assumption and going to the interaction picture through the transformation $U = \exp\left[-i\left(\omega_a a^{\dagger}a + \omega_0\sigma_z/2\right)t\right]$, we end up with the transformed Hamiltonian

$$H_1 = \left[g\,\mathrm{e}^{-i\Delta_a t}\,a + \Omega_1\,\mathrm{e}^{i(\phi_1-\Delta_1 t)} + \Omega_2\,\mathrm{e}^{i(\phi_2-\Delta_2 t)}\right]\sigma_{eg} + \mathrm{H.c.}, \tag{4}$$

where we have defined the detunings $\Delta_a = \omega_a - \omega_0$ and $\Delta_\ell = \omega_\ell - \omega_0$.

Nonadiabatic decoherence-free evolution . To accomplish a decoherence-free evolution of a superposition of the atomic levels, we must first engineer the appropriate interaction between these levels and the cavity mode. To this end we have to adjust the first classical field to resonance with the atomic transition, i.e., $\Delta_1 = 0$. In what follows, we perform two consecutive unitary transformations, first to a framework rotating with frequency Ω_1, $U_1 = \exp\left[-i\Omega_1\left(\mathrm{e}^{i\phi_1}\sigma_{eg} + \mathrm{H.c.}\right)t\right]$, which is straightforwardly done with the help of the basis states $\left\{|\pm\rangle = \left(|e\rangle \pm \mathrm{e}^{-i\phi_1}|g\rangle\right)/\sqrt{2}\right\}$, constituting the eigenstates of the operator $\Omega_1\left(\mathrm{e}^{i\phi_1}\sigma_{eg} + \mathrm{H.c.}\right)$ defining U_1. The adjustment $\Delta_2 = -2\Omega_1$ enables us to proceed to the second transformation $U_2 = \exp\left[-i\Omega_2\left(\mathrm{e}^{i\varphi}\sigma_{+-} + \mathrm{H.c.}\right)t/2\right]$, performed with the help of another set of basis states $\left\{|{}^{\uparrow}_{\downarrow}\rangle = \left(|+\rangle \pm \mathrm{e}^{-i\varphi}|-\rangle\right)/\sqrt{2}\right\}$, composed by eigenstates of the operator $\Omega_2\left(\mathrm{e}^{i\varphi}\sigma_{+-} + \mathrm{H.c.}\right)$. Working in the regime where $\Omega_1 \sim \Delta_2 \gg \Omega_2 \sim \Delta_a \gg g$ and adjusting $\Delta_a = -\Omega_2$, we end up, after a RWA, with the effective Hamiltonian

$$H_2 = (g/2)\left(\mathrm{e}^{i\phi_1}\,a^{\dagger}\sigma_{\uparrow\downarrow} + \mathrm{e}^{-i\phi_1}\,a\sigma_{\downarrow\uparrow}\right). \tag{5}$$

Now, with the engineered interaction (5) and the dissipative mechanisms of both the harmonic mode and the TL system, the evolution of the transformed density operator ρ of the whole system is given by

$$\dot{\rho} = -i\left[H_2,\rho\right] + (\Gamma/2)\left(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a\right) + \mathcal{L}_{TL}\rho,$$

where $\mathcal{L}_{TL}\rho$ stands for the Liouvillian dynamics of the TL system under the transformations $U_{1,2}$ which do not modify the usual Liouvillian form for the harmonic field decay**.** Towards the engineered reservoir, we assume that the decay constant of the cavity field is significantly larger than both the effective coupling $g/2$ and the decay constant γ of the TL system in $\mathcal{L}_{TL}\rho$. In our "cavity QED + trapped ion" system, the regime $\Gamma \gg g$, γ is easily achieved through a cavity with low quality factor $Q = \omega_a/\Gamma$. Together with the good approximation of a reservoir at absolute zero, this regime enables us to consider only the matrix elements $\rho_{mn} = \langle m|\,\rho\,|n\rangle$ inside

the subspace $\{|0\rangle, |1\rangle\}$ of photon numbers. Moreover, following the reasoning in Ref. [8], the strong decay rate Γ enables the adiabatic elimination of the elements ρ_{01} and ρ_{11}, prompting the evolution of the TL system

$$\dot{\rho}_{TL} = \Gamma_{eng}\left(2\sigma_{\uparrow\downarrow}\rho_{TL}\sigma_{\downarrow\uparrow} - \sigma_{\downarrow\downarrow}\rho_{TL} - \rho_{TL}\sigma_{\downarrow\downarrow}\right) + \mathcal{L}_{TL}\rho_{TL}, \tag{6}$$

where $\Gamma_{eng} = g^2/\Gamma$ stands for the coupling strength of the engineered reservoir. The inevitable and undesired action of the multimode vacuum $\mathcal{L}_{TL}\rho$ thus works against the protected state $|\uparrow\rangle\langle\uparrow|$ of the TL system which follows asymptotically from Eq. (6) with $\gamma = 0$. In fact, taking into account the multimode vacuum, the equations of motion of the matrix elements $(\rho_{TL})_{rs} = \langle r|\,\rho\,|s\rangle$ (with r and s standing for $\uparrow$ and $\downarrow$), following from the rotating-wave approximation, are given by

$$\begin{aligned}
\left(\dot{\rho}_{TL}\right)_{\uparrow\uparrow} &= \left(\Gamma_{eng} + 3\gamma/8\right) - \left(\Gamma_{eng} + 6\gamma/4\right)(\rho_{TL})_{\uparrow\uparrow} \\
&= -\left(\dot{\rho}_{TL}\right)_{\downarrow\downarrow}, \\
\left(\dot{\rho}_{TL}\right)_{\uparrow\downarrow} &= -\left(\Gamma_{eng}/2 + 5\gamma/4\right)(\rho_{TL})_{\uparrow\downarrow} + (\gamma/8)(\rho_{TL})_{\downarrow\uparrow} \\
&= \left(\dot{\rho}_{TL}\right)^{*}_{\downarrow\uparrow},
\end{aligned}$$

whose asymptotic solution leads to the protected state

$$\rho_{TL}(t\to\infty) = (1-\varepsilon)\,|\uparrow\rangle\langle\uparrow| + \varepsilon\,|\downarrow\rangle\langle\downarrow|, \tag{7}$$

where $\varepsilon = \left[2 + (8/3)\left(\Gamma_{eng}/\gamma\right)\right]^{-1}$. From Eq. (7), it is straightforward to compute the fidelity of the protected state $|\uparrow\rangle$, given by $\mathcal{F} = \mathrm{Tr}\left[|\uparrow\rangle\langle\uparrow|\,\rho_{TL}(t\to\infty)\right] = 1 - \varepsilon$, which approaches unity for a successfully engineered coupling strength $\Gamma_{eng} \gg \gamma$. It is worth noting that even the modest ratio $\Gamma_{eng}/\gamma = 10$ results in a fidelity around 97%. Considering that the coupling between the ground and excited states is induced by a Raman transition [7], where $g \approx 10^5\ \mathrm{s}^{-1}$ and $\gamma \approx 10^2\ \mathrm{s}^{-1}$ [8, 15], we obtain for a cavity decay constant $\Gamma \approx 10^6\ \mathrm{s}^{-1}$ the strength $\Gamma_{eng}/\gamma \approx 10^2$. Therefore, with the excellent approximation $\varepsilon \ll 1$ and reversing the unitary transformations U_2 and U_1, we note that the state $|\uparrow\rangle$, written in the interaction picture as

$$|\psi(t)\rangle = \cos\left(\varphi/2 - \Omega_1 t\right)|e\rangle - i\,\mathrm{e}^{-i\phi_1}\sin(\varphi/2 - \Omega_1 t)\,|g\rangle, \tag{8}$$

allows for a nonadiabatic coherent evolution of a TL system under spontaneous decay, which can be manipulated through the laser parameters Ω_1, $\varphi = \phi_1 - \phi_2$. Such an evolution corresponds to flips in the atomic states, representing trajectories on different meridian planes on the Bloch sphere, governed by the unitary evolution $\exp\left\{-i\left[(\Omega_1 t - \varphi/2)\left(\sigma_{eg}\,\mathrm{e}^{i\phi_1} + \mathrm{H.c.}\right)\right]\right\}$. At this point we note that the time-independent operator $\sigma_{\uparrow\downarrow}$ corresponds to $\mathcal{O}_R$ mentioned above, while the transformation U_2U_1 corresponds to R.

Quantum memory and adiabatic decoherence-free evolution. To build a quantum memory, a device that protects a stationary superposition, we turn off the second amplification field ($\Omega_2 = 0$) in our starting Hamiltonian (4). Applying the unitary transformation $\widetilde{U}_1 = \exp\left[i\Delta_1\sigma_z/2t\right]$, we thus obtain the time-dependent Hamiltonian $\widetilde{H}_1 = \Delta_1\sigma_z/2 + \Omega_1\left(\mathrm{e}^{i\phi_1}\,\sigma_{eg} + \mathrm{e}^{-i\phi_1}\,\sigma_{ge}\right) + \left[g\,\mathrm{e}^{-i\Delta_a t}\,a\sigma_{eg} + \mathrm{H.c.}\right]$. Under the additional transformation $\widetilde{U}_2 = \exp\left\{-i\left[\Delta_1\sigma_z/2 + \Omega_1\left(\mathrm{e}^{i\phi_1}\,\sigma_{eg} + \mathrm{e}^{-i\phi_1}\,\sigma_{ge}\right)\right]t\right\}$, accomplished with the help of the basis states $\left\{|\widetilde{\pm}\rangle = \left(\sqrt{2\pm\chi}\,|e\rangle \pm \mathrm{e}^{-i\phi_1}\sqrt{2\mp\chi}\,|g\rangle\right)/2\right\}$, with $\chi = \Delta_1/\lambda$, $\lambda = \sqrt{\Omega_1^2 + \Delta_1^2/4}$, and the detuning $\Delta_a = -2\lambda$, we finally obtain the effective interaction

$$\widetilde{H}_2 = (\widetilde{g}/2)\left(\mathrm{e}^{i\phi_1}\,a^{\dagger}\sigma_{+-} + \mathrm{e}^{-i\phi_1}\,a\sigma_{-+}\right), \tag{9}$$

where $\widetilde{g} = g(1-\chi/2)$.

Following the steps outlined above for the addition of the damping mechanism to the cavity mode and the TL system, we reach the master equation $\dot{\widetilde{\rho}}_{TL} = \widetilde{\Gamma}_{eng}\left(2\sigma_{+-}\widetilde{\rho}_{TL}\sigma_{-+} - \sigma_{--}\widetilde{\rho}_{TL} - \widetilde{\rho}_{TL}\sigma_{--}\right) + \widetilde{\mathcal{L}}_{TL}\widetilde{\rho}_{TL}$, with $\widetilde{\Gamma}_{eng} = \widetilde{g}^2/\Gamma$, leading to the asymptotic solution

$$\begin{aligned}
\widetilde{\rho}_{TL}(t\to\infty) = (1-\widetilde{\varepsilon})\,|+\rangle\langle+| + \widetilde{\varepsilon}\,|-\rangle\langle-| \\
+ \widetilde{\varepsilon}(1-\widetilde{\varepsilon})^{-1}\left(|+\rangle\langle-| + |-\rangle\langle+|\right),
\end{aligned}$$

where $\widetilde{\varepsilon} = \left[2 + \left(\widetilde{\Gamma}_{eng}/\gamma\right)\right]^{-1} \ll 1$ for the case where $\widetilde{\Gamma}_{eng} \gg \gamma$, providing again a fidelity ($\mathcal{F} = 1 - \widetilde{\varepsilon}$) around unity for the protected state $|+\rangle$. For the approximation ($\widetilde{\varepsilon} \ll 1$), the state $|+\rangle$ written in the interaction picture is given by

$$\left|\widetilde{\psi}(t)\right\rangle = \frac{\sqrt{2+\chi}\,|e\rangle + \mathrm{e}^{-i(\phi_1 - \Delta_1 t)}\sqrt{2-\chi}\,|g\rangle}{2}. \tag{10}$$

In the resonant case, where $\Delta_1 = \chi = 0$, we obtain the stationary state $|+\rangle = \left(|e\rangle + \mathrm{e}^{-i\phi_1}\,|g\rangle\right)/\sqrt{2}$, which would have been its complementary state $|-\rangle$ if we had set the detuning $\Delta_a = 2\lambda$ or the phase $\phi_1 \to \phi_1 + \pi$. In that case, it is worth noting that if the value of the dephasings changes adiabatically between ϕ_1 and $\phi_1 + 2\pi$, in such a way that the system is always in equilibrium with the engineered reservoir, the Bloch vector representing the protected state (10) performs a complete rotation around the Bloch sphere, as required in Ref. [11], to achieve a coherent evolution of a superposition state driven by an engineered reservoir.

From the above definition, we note that the ratio $\widetilde{\Gamma}_{eng}/\gamma = \left[g(1-\chi/2)\right]^2/(\gamma\Gamma)$ is a function of the parameter χ which defines the polar angle of the state vector (10) on the Bloch sphere. This ratio reaches a maximum when $\Omega_1/\Delta_1 \to 0$ with negative Δ_1 and, consequently, $\chi = -2$, corresponding to the ground state $|g\rangle$. When $\Omega_1/\Delta_1 \to 0$ with positive Δ_1, such that $\chi = 2$, the ratio $\widetilde{\Gamma}_{eng}/\gamma$ is null, forbidding us from protecting the excited state $|e\rangle$. Evidently, the values Δ_1,$\chi \neq 0$ describes nonadiabatic evolution on the Bloch sphere. For

the intermediate value $\chi = 0$, we obtain from the typical strengths considered above for g, γ, and Γ the value $\widetilde{\Gamma}_{eng}/\gamma = 10^2$, representing a fidelity around 99% for the protected 'equatorial' state $|+\rangle$. Whereas the 'equatorial' case $\Delta_1 = \chi = 0$ can be employed to achieve an adiabatic azimuthal evolution of the stationary state $|+\rangle = \left(|e\rangle + \mathrm{e}^{-i\phi_1}|g\rangle\right)/\sqrt{2}$ on the Bloch sphere, the cases $\Delta_1, \chi \neq 0$ describe nonadiabatic evolution on the parallel planes on the Bloch sphere.

Geometric phase induced by reservoir. As can be seen from Eqs. (8) and (10), in the context of decoherence-free evolution, we are able to engineer the non-stationary superposition of atomic states, evolving coherently and acquiring geometric and dynamic phases. In fact, rewriting the state (8) as $|\psi(t)\rangle = \left[|+\rangle + e^{-i(\varphi-2\Omega_1 t)}|-\rangle\right]/\sqrt{2}$, we obtain after a cyclic evolution ($T = \pi/\Omega_1$) the dynamic phase $\phi_D(T) = -\int_0^T \langle\psi(t)|\, H_I(t)\, |\psi(t)\rangle\, dt = -\pi\Omega_2/(2\Omega_1)$ and the geometric one $\phi_G^{cic.}(T) = i\int_0^T \langle\psi(t)|\, \frac{d}{dt}\, |\psi(t)\rangle\, dt = -\pi$, respectively, where $H_I(t) = \Omega_1(\sigma_{++} - \sigma_{--}) + \Omega_2\left[e^{i(\varphi-2\Omega_1 t)}\sigma_{+-} + e^{-i(\varphi-2\Omega_1 t)}\sigma_{-+}\right]/2$. Therefore, considering the total evolution time T, under the regime of parameters stated above ($\Omega_2 \ll \Omega_1$), we see that the contribution coming from the dynamic phase is negligible, while the geometric phase is π.

In order to observe geometric effects we consider an auxiliary atomic level a, which does not couple with the states $|g\rangle$ and $|e\rangle$ through the action of the fields involved in the engineering scheme. We observe that within the time scale T of the experiment, the lifetime of state $|a\rangle$ does not affect the dynamics described by the master equation (6). Otherwise the level a can be chosen as a more excited metastable state. To measure the phases acquired by the state $|\psi(t)\rangle$, we must employ an interferometric scheme with $|a\rangle$ as the reference state [11]. For this purpose, using the conservation of the total probability, $\rho_{aa}^I(t) + \rho_{++}^I(t) + \rho_{--}^I(t) = 1$, we solve the system of coupled differential equations for the probability amplitudes $\rho_{ij}^I(t)$ $(i, j = +, -, a)$ following from Eq. (6) in the interaction picture, disregarding, within the evolution time T, the small contribution of $\mathcal{L}_{TL}$. Supposing now that the initial state of the system is $|\Psi_I(0)\rangle = (|\psi(0)\rangle + |a\rangle)/\sqrt{2}$,with $\phi_1 = \phi_2 = 0$, we find at time t that $\rho^I(t) = |\Psi_I(t)\rangle\langle\Psi_I(t)|$, where $|\Psi_I(t)\rangle = \left(|a\rangle + e^{-i(\Omega_1+\Omega_2/2)t}|\psi(t)\rangle\right)/\sqrt{2}$. As we have set $\phi_1 = \phi_2 = 0$, the protected state at $t = 0$ turns out to be $|\psi(0)\rangle = |e\rangle$. For this reason, the superposition state $|\Psi_I(0)\rangle$ may be obtained by applying a laser pulse between the states $|a\rangle$ and $|e\rangle$. Finally, the geometric phase may be observed through the population inversion $P_{ea}(t) = \cos\left[(2\Omega_1 + \Omega_2)\, t\right]/2 \simeq \cos\left[2\phi_G^{cic.}(t)\right]/2$, where $t = n\pi/\Omega_1; n \in \mathbb{N}$.

It is worth noting that, differently from the scheme proposed in Ref. [11], where the superposition $|\Psi_I(t)\rangle$, used to measure the geometric phase, is affected by decoherence under a nonadiabatic evolution, here $|\Psi(t)\rangle$ is unaffected by the reservoir even under such an evolution faster than that determined by the time scale of the engineered reservoir.

Summarizing, we have improved the engineering reservoir by producing a time-dependent master equation leading to a nonstationary superposition equilibrium state that can be nonadiabatically controlled by the system-reservoir parameters. Working with an ion trapped inside a bad cavity we constructed two classes of decoherence-free evolution of the ground and excited ionic levels. By combining the two classes of evolution, we can manipulate trajectories on the Bloch sphere by changing, alternately, the polar and azimuthal angles. Although in our schemes the protected states acquire dynamic phases, this fact is unimportant, since they remain in a decoherence-free subspace, where the dynamic and geometric phases are unaffected by the reservoir. Finally, we have also generalized the objective of the Refs. [11, 14], showing how to observe the geometric phases acquired by the protected nonstationary states even under a nonadiabatic evolution. We believe that the extention of the present scheme for the nonadiabatic time-dependent control of a set of qubits, generating quantum logic operations inside decoherence-free subspaces, may improve quantum computation.

We wish to express our thanks to the UFABC and for the support from FAPESP, CAPES, and CNPq, Brazilian agencies.

[1] R. Landauer, Proc. R. Soc. London, Ser. A, **353**, 367 (1995); W. G. Unruh, Phys. Rev. A **51**, 992 (1995).
[2] P.W. Shor, Phys. Rev. A **52**, R2493 (1995); A. M. Steane, Phys. Rev. Lett. **77**, 793 (1996), For a review see, e.g., A. M. Steane, in Introduction to Quantum Computation and Information, edited by H. K. Lo, S. Popescu, and T. P. Spiller (Word Scientific, Singapore, 1999), pp. 184-212.
[3] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997); D. A. Lidar *et al.*, Phys. Rev. Lett. **81**, 2594 (1998); E. Knill *et al.*, *ibid* **84**, 2525 (2000); A. R. Bosco de Magalhães and M. C. Nemes, Phys. Rev. A **70**, 053825 (2004); D. A. Lidar and K. B. Whaley, quant-ph/0301032; M. A. de Ponte *et al.*, Annals of Physics **322**, 2077 (2007).
[4] G. S. Agarwal, Phys. Rev. A **61**, 013809 (1999); L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999); D. Vitali and P. Tombesi, Phys. Rev. A **59**, 4178 (1999); L. Viola and E. Knill, Phys. Rev. Lett. **94**, 060502 (2005); F. F. Fanchini, J. E. Hornos, and R. d. J. Napolitano, Phys. Rev. A **75**, 022329 (2007).
[5] L. C. Céleri *et al.*, quant-ph/0609199.
[6] J. F. Poyatos *et al.*, Phys. Rev. Lett. **77**, 4728 (1996).
[7] C. J. Myatt *et al.*, Nature **403**, 269 (2000).
[8] A. R. R. Carvalho *et al.*, Phys. Rev. Lett. **86**, 4988 (2001).
[9] N. Lutkenhaus *et al.*, Phys. Rev. A **57**, 548 (1998); S.

G. Clark and A. S Parkins, Phys. Rev. Lett. **90**, 047905 (2003).
[10] F. O. Prado *et al.*, Phys. Rev. A **73**, 043803 (2006); R. M. Serra *et al.*, Phys. Rev. A **71**, 045802 (2005); C. J. Villas-Bôas and M. H. Y. Moussa, Euro. Phys. Journal D **32**, 147 (2005).
[11] A. Carollo, G. M. Palma, A. Lozinski, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **96**, 150403 (2006).
[12] A. Carollo and G. M. Palma, Las. Phys. **16**, 1595 (2006).
[13] Z. Yin , F. Li, and P. Peng, Phys. Rev A **76**, 062311 (2007).
[14] A. Carollo, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **96**, 020403 (2006).
[15] R. M. Serra *et al.*, Phys. Rev. A **64**, 033419 (2001).